Why do measurements need a ± value?

A ruler has a limit

Imagine measuring a leaf. Its tip falls between two ruler marks. You can estimate its length, but the ruler cannot show every tiny difference.

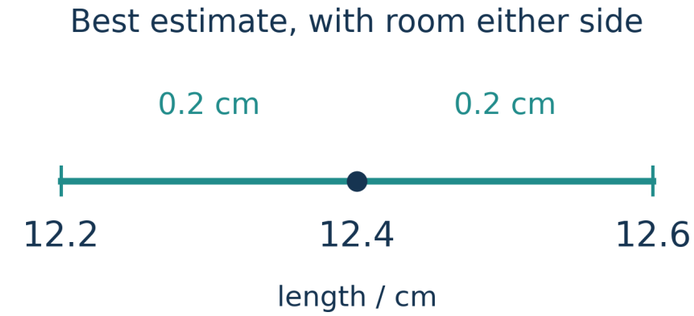


Two pieces of information

12.4 cm is your best estimate. ±0.2 cm says how much uncertainty you have allowed around it. This helps someone judge how closely the length is known.

Why it matters

If two leaves differ by only 0.1 cm, this measuring method may not distinguish their lengths clearly. Uncertainty helps us decide what the evidence can support.



$$L = (12.4 \pm 0.2)\ \text{cm}$$

Uncertainty is part of an honest measurement, not a sign of poor work.

What does ±0.2 actually mean?

Start at the measured value

For 12.4 ± 0.2 cm, subtract 0.2 to find the lower end: 12.2 cm. Add 0.2 to find the upper end: 12.6 cm.

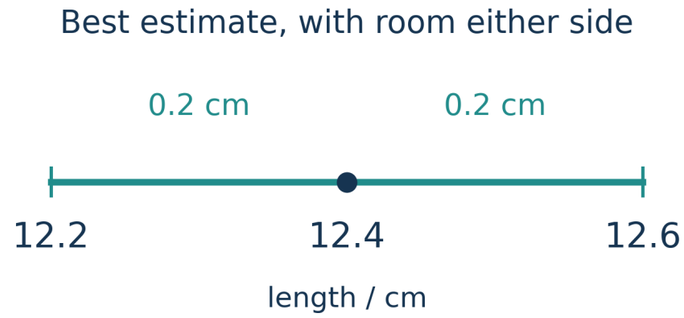


Read it in words

“My best estimate is 12.4 cm, with a stated uncertainty of 0.2 cm either way.” The whole interval is 0.4 cm wide.

An estimate has limits

This interval reflects the uncertainty you included. It is not a guarantee that every possible source of error has been covered. It does not automatically mean 95% confidence.



$$12.4 - 0.2 = 12.2$$

$$12.4 + 0.2 = 12.6$$

The number after ± is the distance from the centre to one end.

Where does the uncertainty come from?



A marked scale

For a thermometer marked every 1 °C, a common classroom reading estimate is ±0.5 °C. You judge the reading between the marks.

A digital display

For a balance displaying 0.1 g steps, a common classroom estimate is ±0.1 g. More decimal places let you see smaller changes.

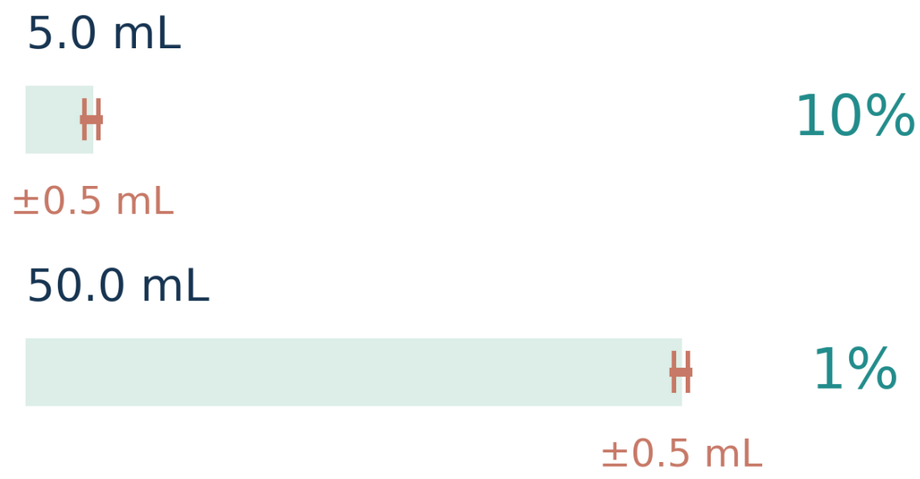
Check the actual instrument

Use a stated uncertainty or manufacturer specification when provided. These classroom estimates describe reading limits. Calibration, unstable readings and technique can add other effects.

A finer display helps, but it does not guarantee a correct reading.

Why turn uncertainty into a percentage?





The same ± can matter differently

An uncertainty of ±0.5 mL is a large part of 5.0 mL, but only a small part of 50.0 mL. The percentage makes that comparison fair.

Find the fraction first

Divide the absolute uncertainty by the measured value. For 5.0 mL: 0.5 ÷ 5.0 = 0.10. Multiply by 100 to get 10%.

Keep the meanings clear

Absolute uncertainty has the same unit as the measurement. Fractional uncertainty has no unit. Percentage uncertainty is the fraction written out of 100.

$$\% \text{ uncertainty} = \frac{\text{absolute uncertainty}}{|\text{measured value}|} \times 100$$

Ask: how large is the uncertainty compared with what I measured?

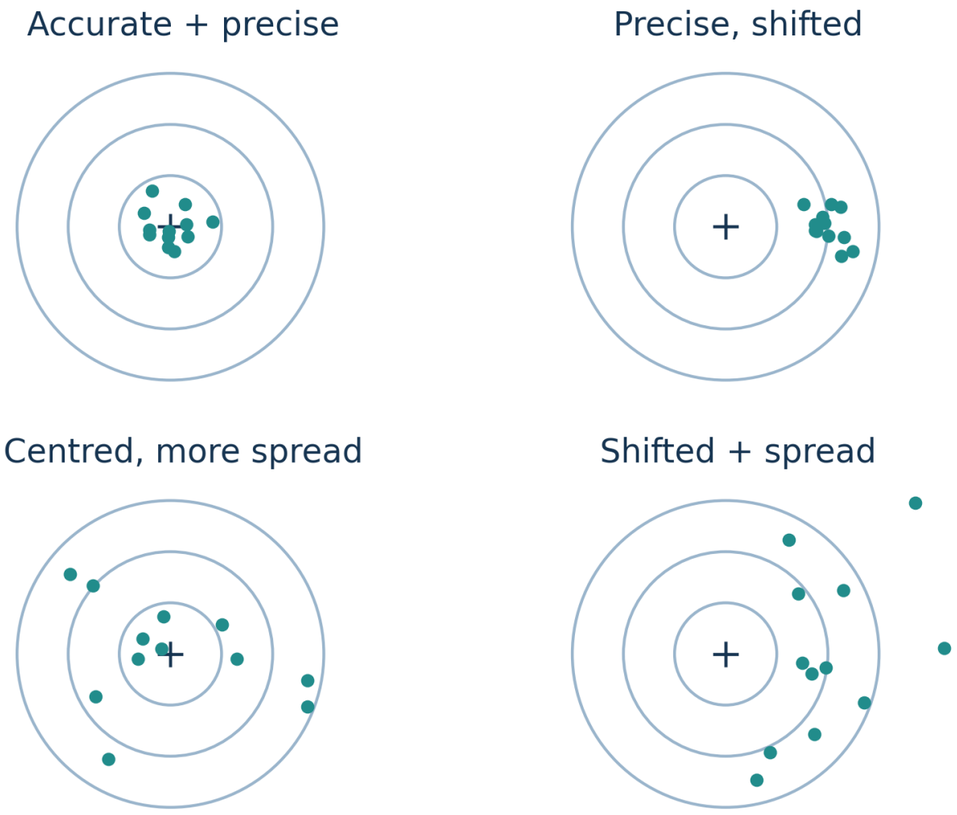
Close together, or close to the target?

Precision means agreement

Imagine repeating the same measurement. Results close together have high precision. They show little spread.

Accuracy means closeness

A result is accurate when it is close to the true value. In practice, we often compare with a suitable accepted reference.



Why we need both

A balance that always reads too high can give a tight group of results in the wrong place. Repeating its readings does not repair its calibration.

A tight group can still miss the target.

Why do results scatter or shift?

Random effects create scatter

Small changes in reaction time make stopwatch readings vary. Some readings are higher and some lower. Repeats help reveal this variation.

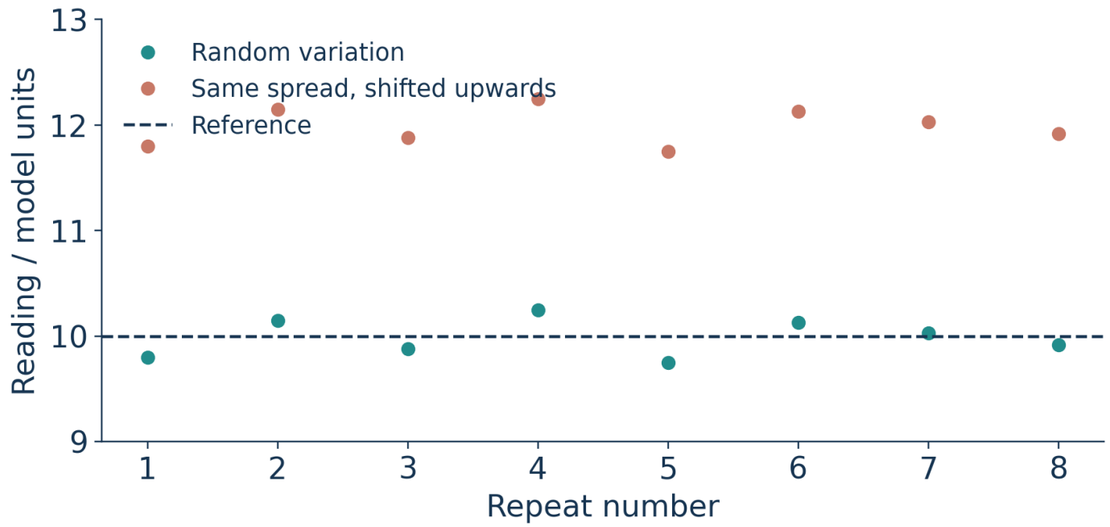


Systematic effects create bias

A balance with a stable +5 g offset shifts all readings upwards. Heat escaping from a food calorimeter tends to make the measured energy too low.

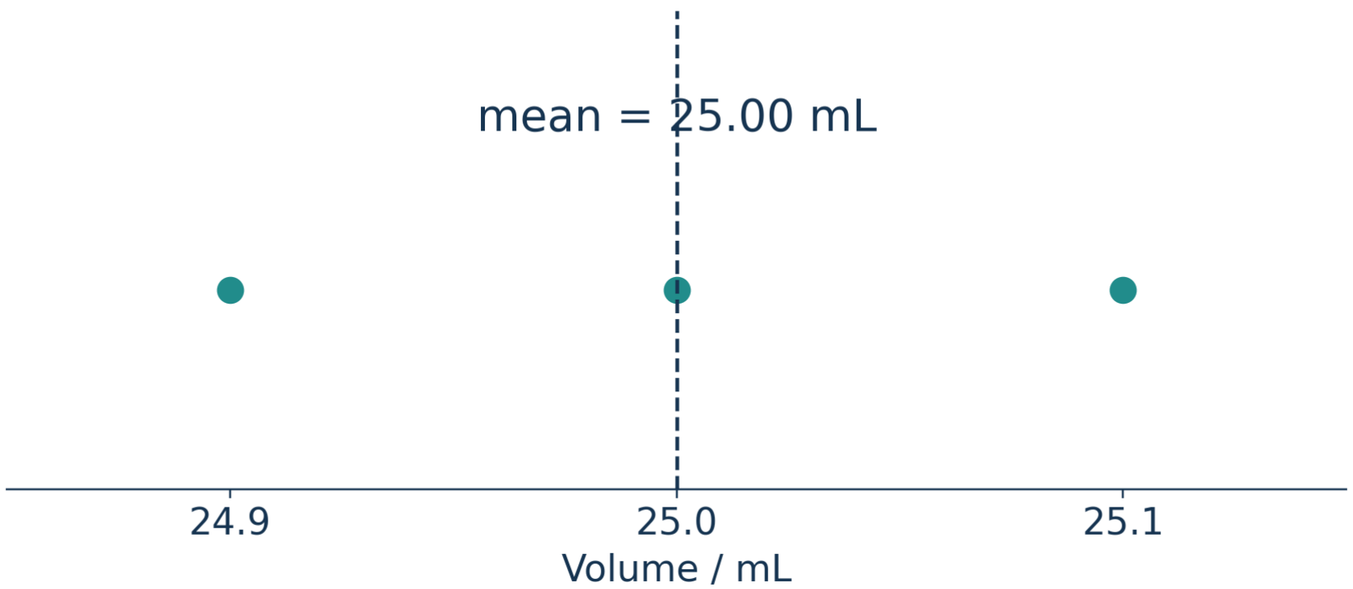
Choose a useful response

Average repeated trials to reduce random variation in the mean. Check calibration or change the method to reduce an identified bias. A ± value can include several kinds of uncertainty.



Repeating a measurement and improving the method solve different problems.

Why repeat a measurement?



$$\text{mean} = \frac{24.90 + 25.00 + 25.10}{3} = 25.00 \text{ mL}$$

One reading may be unusually high

Three volume readings are 24.90, 25.00 and 25.10 mL. Their mean is 25.00 mL. High and low random deviations can partly balance out.

Keep the spread visible

The range runs from 24.90 to 25.10 mL. Half the range is 0.10 mL. A classroom spread estimate is therefore 25.00 ± 0.10 mL.

Repeats have limits

Three repeats give only a rough picture of variation. An instrument offset remains in the mean. Do not divide an instrument uncertainty by the number of repeats.

A mean uses all the readings. The spread tells you how much they varied.

What do SD and SE tell us?

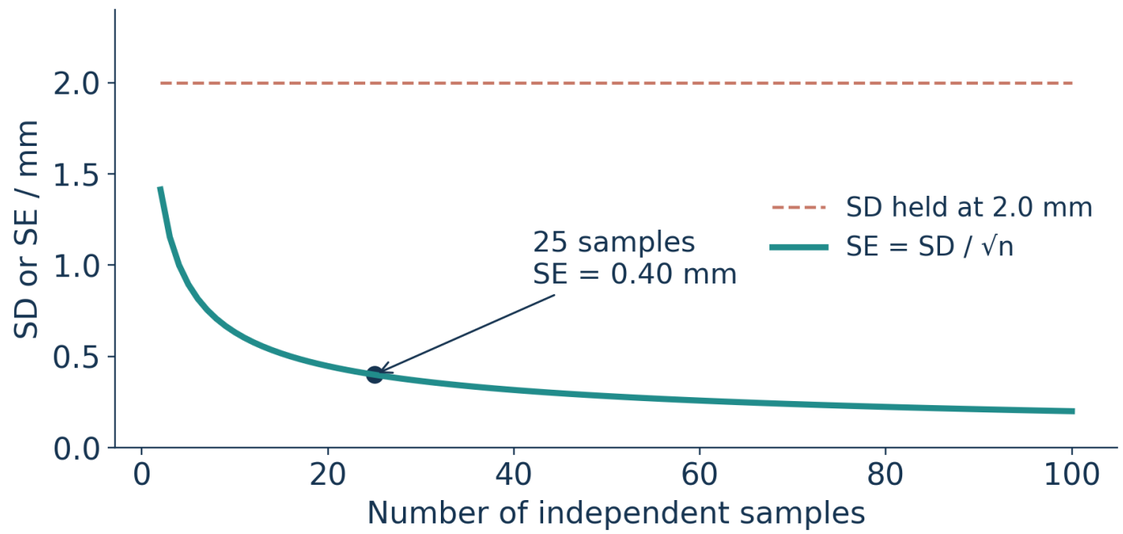
SD describes the individual results

Standard deviation, or SD, describes how spread out the measurements are. A large SD could reflect real differences between organisms.

SE describes the estimated mean

Standard error, or SE, describes how uncertain the sample mean is as an estimate of the population mean. With more independent samples, the mean usually becomes better known.

$$SE = \frac{SD}{\sqrt{n}}$$

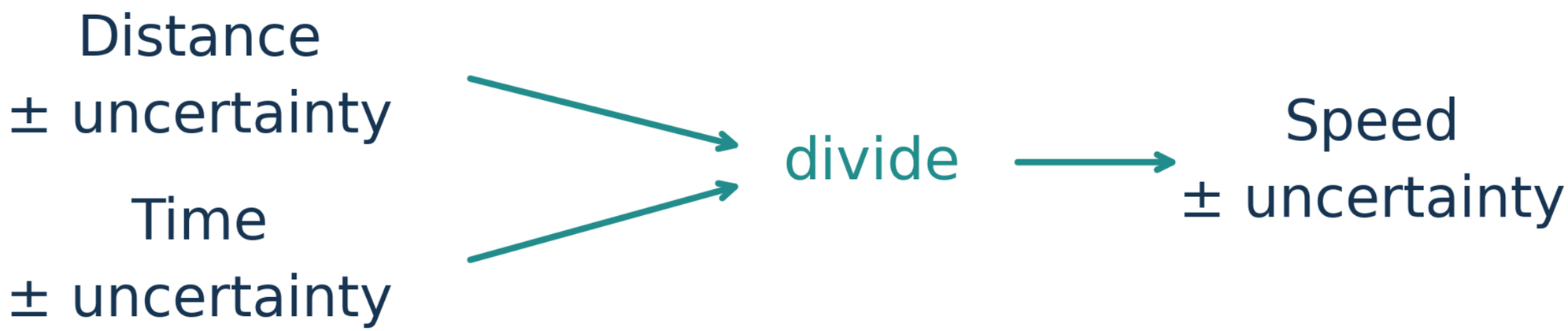


Why the labels matter

If SD = 2.0 mm and there are 25 independent samples, SE = 2.0 ÷ √25 = 0.40 mm. Small SE does not mean every organism is similar. Say which measure your error bars show.

SD: how spread out are the results? SE: how well do they locate the mean?

Why does uncertainty travel through a calculation?



The answer uses measured inputs

You calculate a speed from distance and time. If either measurement changes slightly, the calculated speed changes too.

Carry the uncertainty forward

Propagation means working out how uncertainty in the inputs affects the answer. A calculator cannot make uncertain measurements exact.

Two calculations run together

First calculate the value. Then calculate its uncertainty using the same measured inputs. Finally report the answer and its ± value together. The next cards explain why the rules work.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

The answer inherits uncertainty from the measurements used to calculate it.

Why add uncertainties when subtracting?

Find the temperature rise

The water starts at 24.3 ± 0.1 °C and finishes at 36.7 ± 0.1 °C. The measured rise is 36.7 − 24.3 = 12.4 °C.

Smallest rise $36.6 - 24.4 = 12.2\ °C$

Measured rise $36.7 - 24.3 = 12.4\ °C$

Imagine the widest possible rise

Use a high final value and a low initial value: 36.8 − 24.2 = 12.6 °C. The smallest rise is 36.6 − 24.4 = 12.2 °C.

Largest rise $36.8 - 24.2 = 12.6\ °C$

$$0.1 + 0.1 = 0.2\ ^\circ C$$

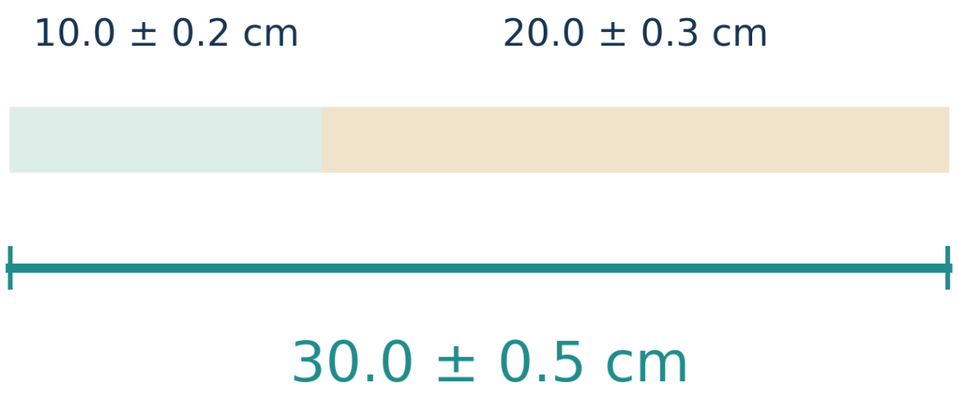
$$\text{rise} = (12.4 \pm 0.2)\ ^\circ C$$

Both readings affect the gap

The result can move 0.2 °C either way. So the classroom rule adds the two reading uncertainties, even though it subtracts the temperatures.

Subtract the readings. Add their absolute uncertainties.

Why add uncertainties when adding?



$0.2 + 0.3 = 0.5$ cm

Imagine joining two strips

One is 10.0 ± 0.2 cm long. The other is 20.0 ± 0.3 cm. Their total measured length is 30.0 cm.

Let both be longer

The upper lengths give 10.2 + 20.3 = 30.5 cm. The lower lengths give 9.8 + 19.7 = 29.5 cm. Both uncertainties can move the total in the same direction.

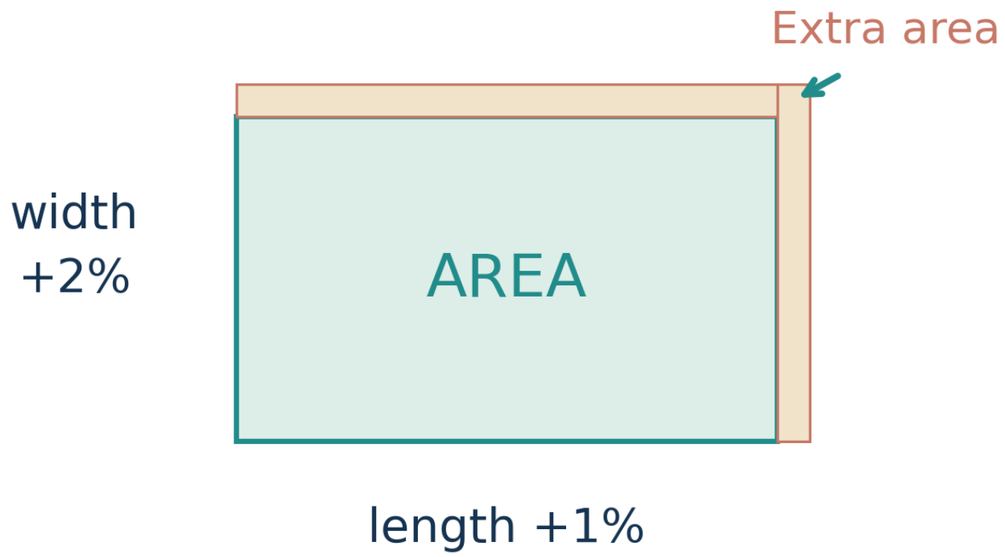
Use matching units

The total is 30.0 ± 0.5 cm. Adding absolute uncertainties works because both contributions describe lengths in the same unit.

For a sum or a difference, add absolute uncertainties in matching units.

Why use percentages when multiplying?

Change exaggerated to show the idea



$1\% + 2\% \approx 3\%$

Think about the area of a rectangle

Area = length × width. A 1% increase in length makes the area 1% larger if width stays the same. A change in width also changes the area.

Let both dimensions increase

If length rises by 1% and width by 2%, area changes by 1.01 × 1.02 = 1.0302. That is 3.02%, very close to 1% + 2% = 3%.

This explains the classroom rule

For small uncertainties, add percentage uncertainties when multiplying. This allows for contributions acting together. It is an approximation, so keep extra digits until the end.

Multiplication combines relative changes, so we use fractions or percentages.

Why does division also use percentages?

Start with density

Mass = 20.0 ± 0.1 g and volume = 10.0 ± 0.2 cm³.
Density = 20.0 ÷ 10.0 = 2.00 g cm⁻³.

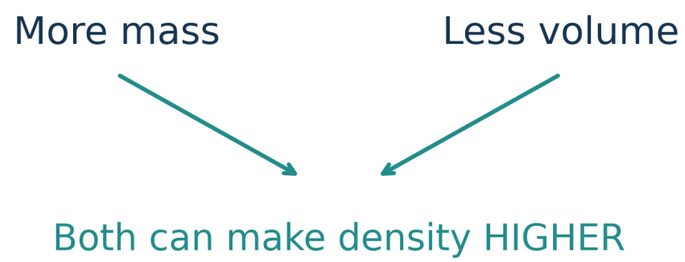


Picture what makes density larger

A higher mass increases density. A lower volume also increases it. Their effects can add, even though the calculation divides.

Put both effects on the same scale

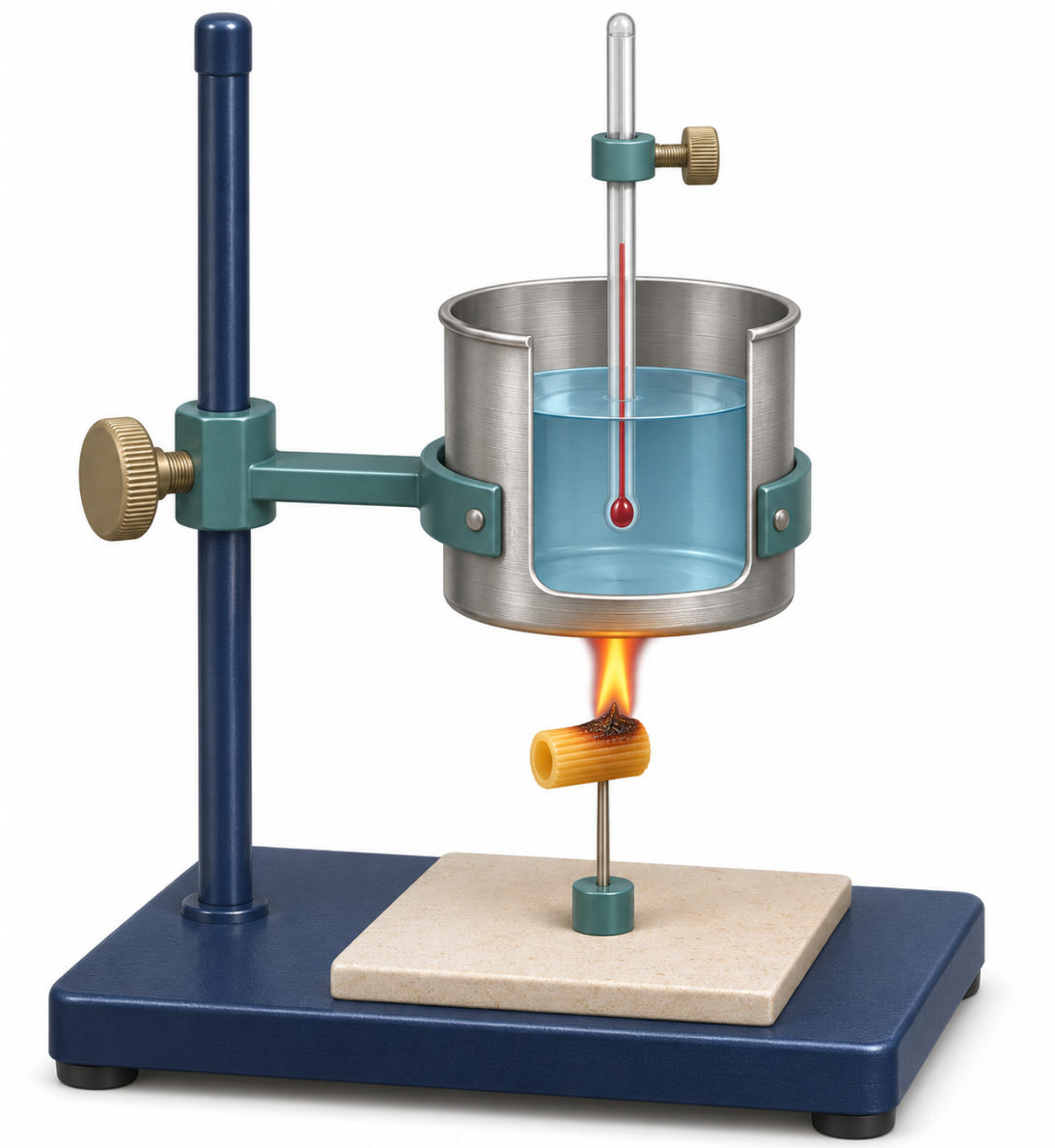
Mass uncertainty = 0.5%. Volume uncertainty = 2.0%. Add them to get about 2.5%. Then 2.5% of 2.00 is 0.05 g cm⁻³.



$$\rho = (2.00 \pm 0.05)\ \mathrm{g\,cm^{-3}}$$

Do not add grams to cubic centimetres. Compare their percentage effects.

The pasta investigation: what are we finding?



The question

How much energy can we transfer to water for each gram of pasta burned? We follow this question from the raw readings to the conclusion.

The water is our energy detector

Burning pasta warms the water. More water, or a larger temperature rise, means more energy has reached the water.

The important limitation

Some energy warms the can or escapes to the surroundings. The water therefore captures only part of the released energy. This setup does not directly measure all the energy in the pasta.

Follow one story: water warmed, energy transferred, energy per gram.

Why weigh the can twice?

We need the mass of water

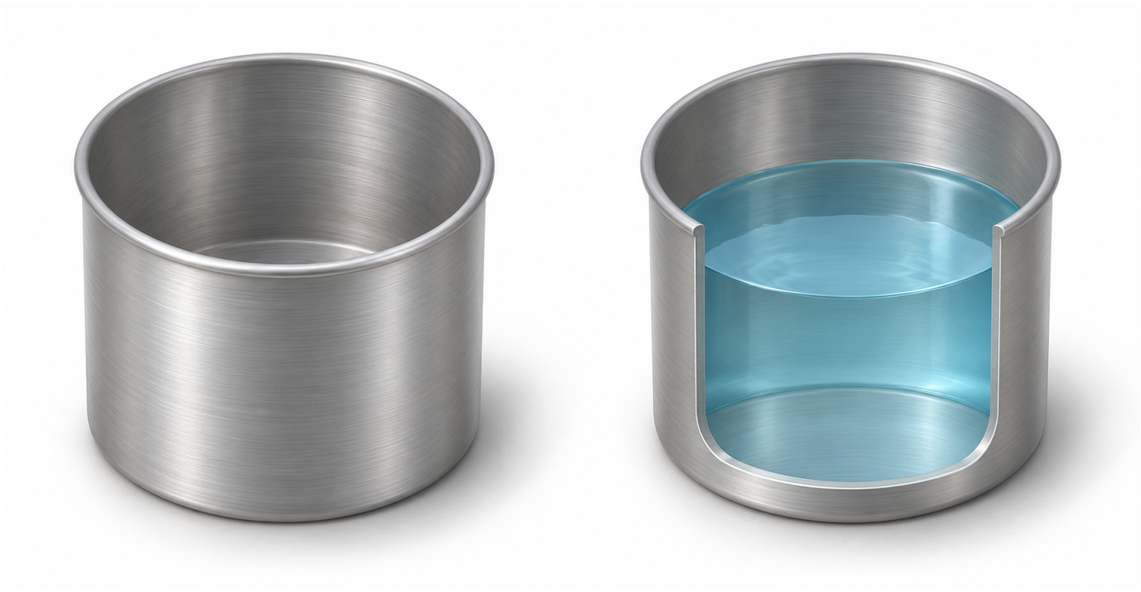
The balance first measures the empty can, then the can with water. Subtracting removes the mass of the container.

Use the measured values

Can with water: 135.801 ± 0.001 g. Empty can: 25.822 ± 0.001 g. Water mass = 109.979 g.

Both readings contribute

Using the classroom difference rule, 0.001 + 0.001 = 0.002 g. The water mass is therefore 109.979 ± 0.002 g. Keep this uncertainty with the value.



Empty can

25.822 g

Can + water

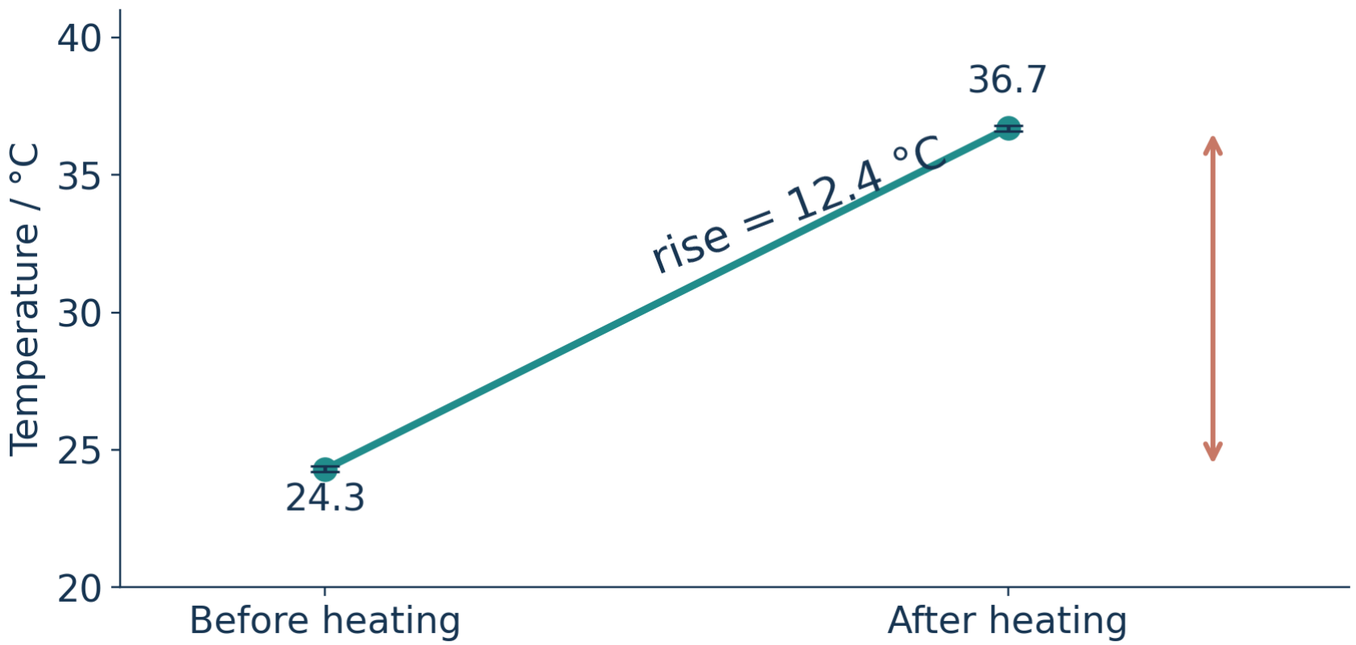
135.801 g

Water adds the difference in mass.

$$135.801 - 25.822 = 109.979 \text{ g}$$

A measured difference carries uncertainty from both readings.

Why use the temperature rise?



$$\Delta T = 36.7 - 24.3 = 12.4 \ ^\circ\text{C}$$

The water was already warm

A final temperature of 36.7 °C does not tell us how much warming occurred. We need to compare it with the starting temperature.

Calculate the change

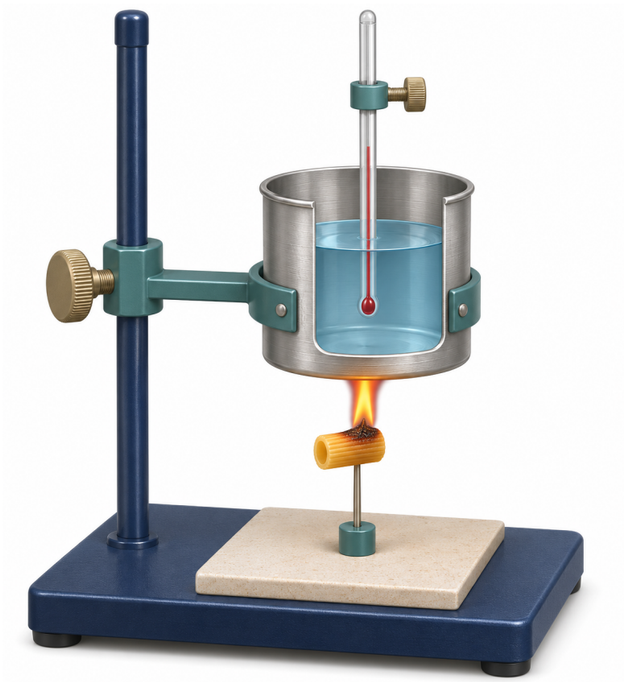
The water warms from 24.3 to 36.7 °C. The rise is 12.4 °C. Each reading has ±0.1 °C uncertainty, so the rise has ±0.2 °C.

Give the symbol a meaning

We write the temperature change as ΔT. Here Δ means “change”. The uncertainty in that change is the separate ±0.2 °C value.

The energy calculation needs how much the water warmed, not its final temperature.

Why multiply mass × heat capacity × rise?



109.979 g × 4.18 J per g per °C × 12.4 °C

how much water | energy for each gram and each degree | how much warming

= 5700.43 J gained by the water

Start with one gram and one degree

Water needs about 4.18 J to warm 1 g by 1 °C. This is its specific heat capacity, $c = 4.18\ J\ g^{-1}\ °C^{-1}$.

Scale up to our water

We have 109.979 g, and each gram warms by 12.4 °C. Multiply all three numbers: $q = mc\Delta T = 5700.431528$ J.

Name the answer correctly

q is the energy gained by the water. It is positive because the water warms. We treat c as exact for this classroom uncertainty calculation. Real water properties and heat losses are separate considerations.

$$q = 109.979 \times 4.18 \times 12.4 \approx 5700.43\ \text{J}$$

More grams need more energy. More degrees of warming also need more energy.

Why subtract the leftover pasta?

Some pasta remains

The starting piece is 1.149 ± 0.001 g. The remaining material is 0.053 ± 0.001 g. The decrease in mass is 1.096 g.

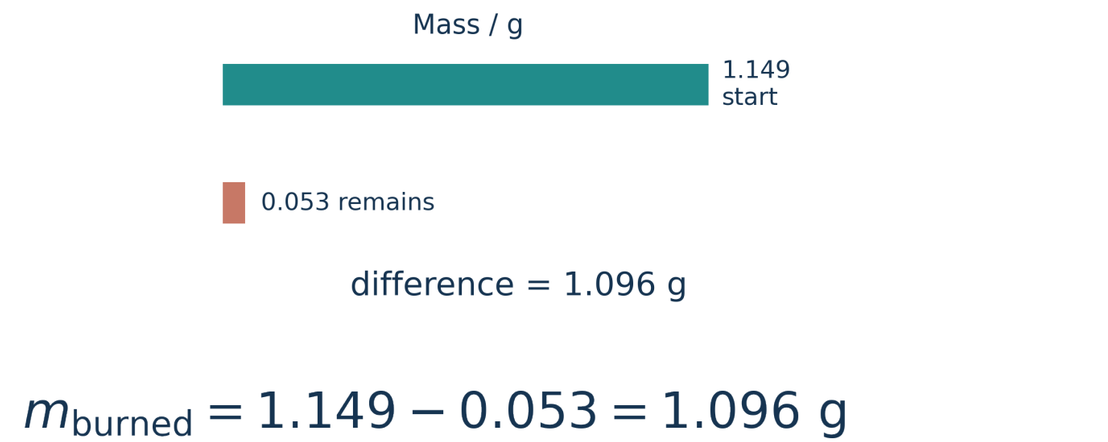


Use the difference rule again

Add the absolute uncertainties: 0.001 + 0.001 = 0.002 g. We use 1.096 ± 0.002 g as the estimated mass burned.

Check the assumption

This treats the measured mass loss as fuel burned. Lost fragments, moisture loss or incomplete combustion can make that estimate imperfect. The balance uncertainty alone does not cover these effects.



$$m_{\text{burned}} = 1.149 - 0.053 = 1.096 \text{ g}$$

We want energy per gram burned, not per gram originally present.

Why divide by the mass burned?

A larger piece can release more energy

Comparing total energy from different-sized pieces would be unfair. “Per gram” gives a common basis for comparison.

Share the energy across the grams

Our 1.096 g of pasta transferred 5700.43 J to the water. Dividing gives about 5201.12 J for each gram burned.

Read the units aloud

J g^{-1} means joules per gram. We can also write 5.20112 kJ g^{-1} because 1000 J = 1 kJ. Keep these extra digits for the uncertainty calculation.



Before burning

Remaining material

5700.43 J spread across 1.096 g

↓

divide by 1.096

↓

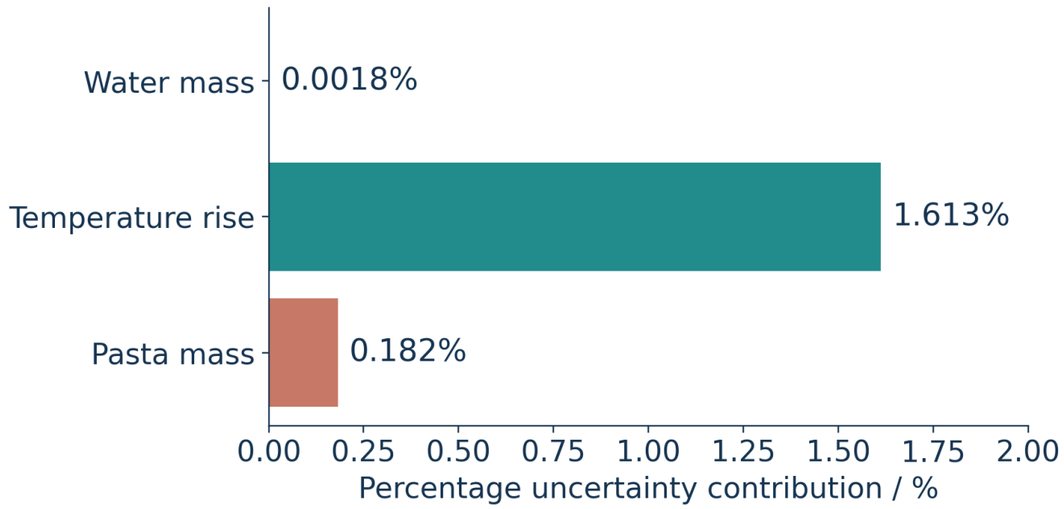
5201.12 J for each gram

$$E = \frac{q}{m_{\text{burned}}} = \frac{5700.431528}{1.096}$$

$$E \approx 5201.12\ \text{J g}^{-1}$$

“Per gram” means divide by the number of grams.

Which uncertainties affect energy per gram?



Look at the whole calculation

Energy per gram = water mass × heat capacity × temperature rise ÷ pasta mass burned. Three measured inputs carry stated uncertainties here.

Convert each to a percentage

Water: 0.002 ÷ 109.979 × 100 = 0.0018%. Rise: 0.2 ÷ 12.4 × 100 = 1.613%. Pasta: 0.002 ÷ 1.096 × 100 = 0.1825%.

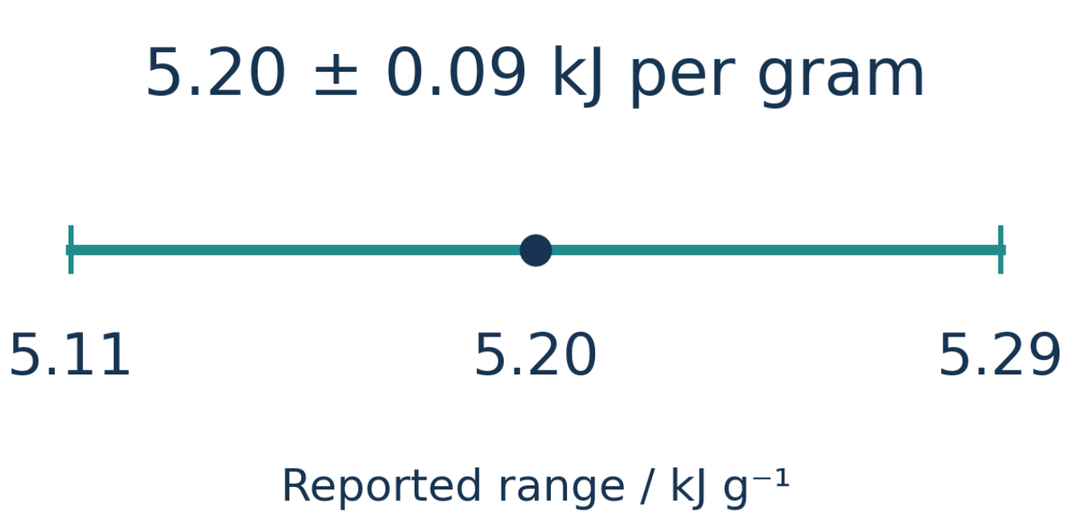
$$0.0018\% + 1.613\% + 0.1825\% \approx 1.797\%$$

Add their percentage effects

The calculation uses multiplication and division, so add these contributions. Total percentage uncertainty ≈ 1.797%, or about 1.8%. The temperature rise contributes most.

First find each uncertainty as a fraction of its own measured value.

How does 1.8% become a ± value?



$$E = (5.20 \pm 0.09)\ \text{kJ g}^{-1}$$

Undo the percentage calculation

1.797% means about 1.797 parts in every 100. Divide by 100, then multiply by the energy-per-gram result.

Calculate the absolute uncertainty

5201.12 × 0.01797 ≈ 93.5 J g^{-1}. Using one significant figure for this uncertainty gives ±90 J g^{-1}.

Match the final decimal place

Round the value to the nearest 10 J g^{-1} too. Report (5.20 ± 0.09) × 10^3 J g^{-1}, or 5.20 ± 0.09 kJ g^{-1}. Other stated rounding conventions may retain two uncertainty digits.

Keep extra digits during the working. Round the final answer and uncertainty together.

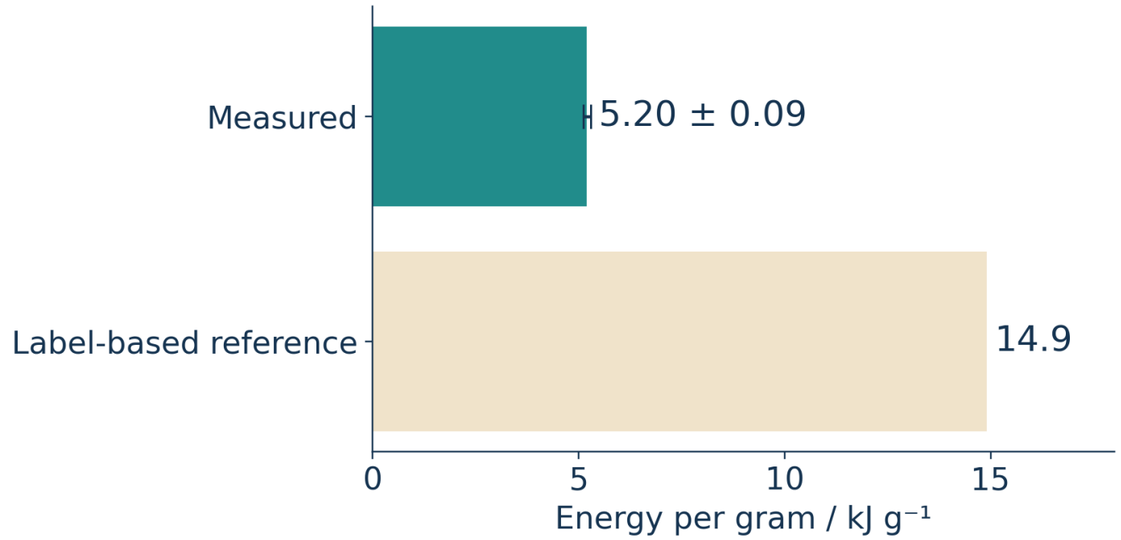
How far is our answer from the label?

Compare the same units

The package data give 837 kJ per 56 g. That is about 14.9 kJ g^{-1}. Our measured value is about 5.20 kJ g^{-1}.

Calculate percentage error

The gap is 14.9 − 5.20 = 9.70 kJ g^{-1}. Divide by the reference value, then multiply by 100. The discrepancy is about 65%.



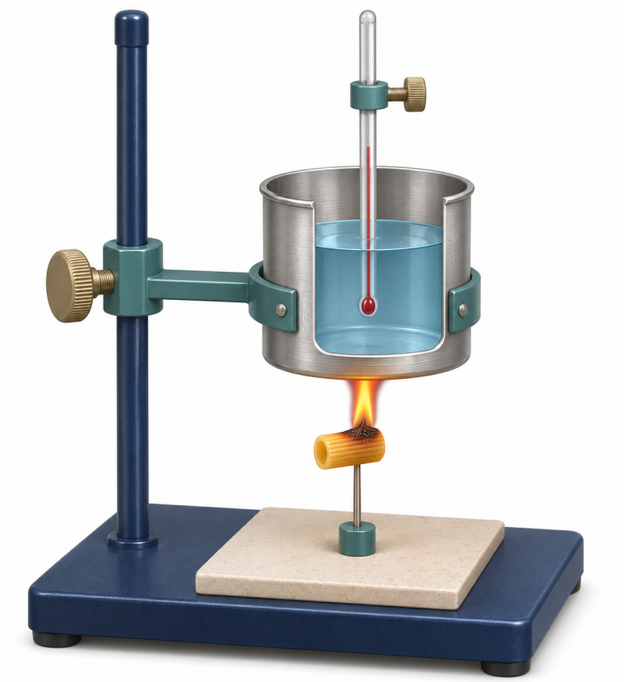
$$\% \text{ error} = \frac{|5.20 - 14.9|}{14.9} \times 100 \approx 65\%$$

Treat the comparison fairly

A food label is a useful reference, not an exact combustion standard. Nutritional energy and this simple heat-transfer experiment differ. A food Calorie means a kilocalorie: 200 kcal is about 837 kJ.

Percentage error compares with a reference. Percentage uncertainty describes our result.

Why can a small ± sit beside a big error?



Energy released by burning pasta

- Warms the water counted by $q = mc\Delta T$
- Warms the metal can
- Escapes to the surroundings

The ± covers the stated readings

About 1.8% uncertainty comes from the balance and thermometer values included in our calculation. It does not include every way the method can lose energy.

Follow the missing energy

Some heats the metal can. Some warms the surrounding air. Incomplete burning can leave chemical energy unreleased. These effects help explain a low measured energy per gram.

What we can conclude

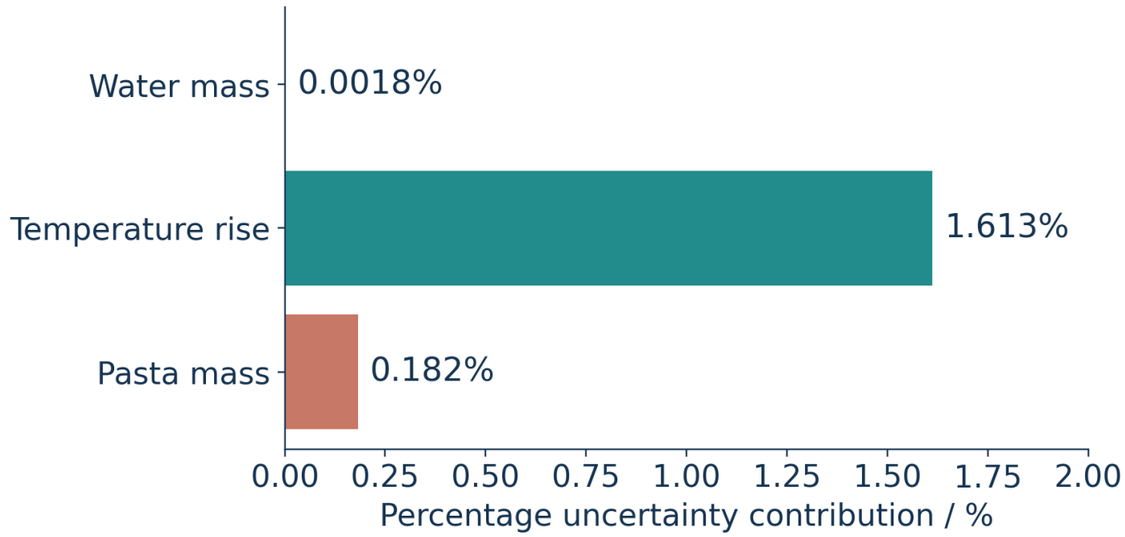
The gap from the label is much larger than the included reading uncertainty. Investigate omitted effects and the reference comparison. These numbers alone do not prove repeatability or identify one exact cause.

Small reading uncertainty cannot repair a method that misses part of the energy.

Which improvement would help most?

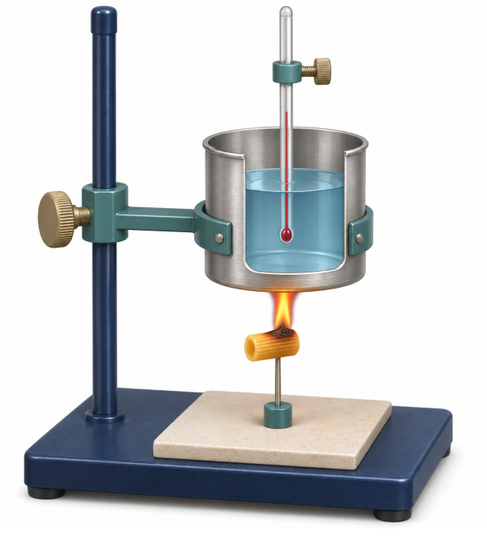
For the calculated uncertainty

Temperature contributes about 1.61%, much more than either mass term. A suitable finer temperature sensor would reduce this contribution more than a finer balance.



For the low energy result

Use an appropriate heat shield and account for energy warming the can. Improve combustion where possible. Each change targets a specific reason that the water receives too little energy.



Explain the benefit

Avoid "use better equipment". Say which measurement or energy loss improves and why. Repeating trials helps assess variation, but does not recover energy that escaped in every trial.

temperature : 1.613% pasta : 0.1825%

Improve the largest uncertainty contribution, and address the method's bias separately.

What would a clear conclusion say?

RESULT

5.20 ± 0.09 kJ per gram

↓

What did we actually measure?

Energy that reached the water

State the result

"The experiment transferred 5.20 ± 0.09 kJ to the water per gram of pasta burned, using the stated reading uncertainties."

Explain the comparison

"This is about 65% below the label-based reference of 14.9 kJ g^{-1}. The included reading uncertainty is too small to explain that gap."

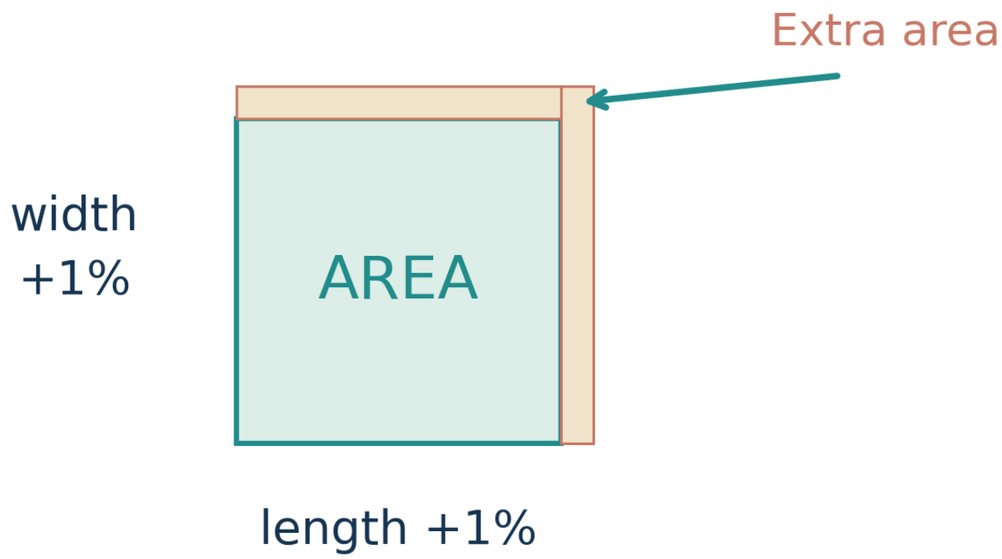
Explain the limitation

"Heat loss and energy warming the can could lower the result. The label is also an imperfect comparison. Repeated trials are needed to judge repeatability." Do not calculate "systematic error" by subtracting two percentages.

A conclusion links the numbers to what the experiment actually measured.

Why do powers multiply percentage uncertainty?

Change exaggerated to show the idea



$A = (12.57 \pm 0.25)\ \text{cm}^2$

A square uses the same length twice

For area $A = l^2$, increasing l by 1% increases both factors. The area becomes $1.01 \times 1.01 = 1.0201$ times its original value, about 2% higher.

Use the power as the multiplier

For small uncertainties, a square doubles the percentage uncertainty. A cube triples it. A square root halves it. Exact constants such as π add no uncertainty.

Try a circular area

Radius = 2.00 ± 0.02 cm, so radius uncertainty = 1%. Area = $\pi r^2 \approx 12.57\ \text{cm}^2$. Area uncertainty ≈ 2%, or $0.25\ \text{cm}^2$.

The power tells you how strongly a small relative change affects the answer.

Why time several swings?

Reaction time affects the stopwatch

Suppose 20 complete swings take 40.0 ± 0.2 s. The fixed timing uncertainty is only 0.5% of that long interval.

Divide by an exact count

One swing takes 40.0 ÷ 20 = 2.00 s. Divide the absolute uncertainty by 20 too: 0.2 ÷ 20 = 0.01 s. The result is 2.00 ± 0.01 s.



$$T = (2.00 \pm 0.01)\ \text{s}$$

Why this helps

Timing one 2.00 s swing with the same ±0.2 s would give 10% uncertainty. Timing many swings makes reaction time a smaller fraction. Count carefully and use the same start/finish position.

An exact divisor scales the value and absolute uncertainty by the same amount.

The same thinking works in other labs

Burette readings give a difference

Initial = 1.20 ± 0.05 mL. Final = 24.60 ± 0.05 mL. Delivered volume = 23.40 ± 0.10 mL. Both readings matter because you subtract them.

Gas production gives a rate

A sample produces 24.0 ± 1.0 mL in 60.0 ± 0.5 s. Rate = 24.0 ÷ 60.0 = 0.400 mL s^{-1}. Add percentages: 4.17% + 0.83% = 5.0%.



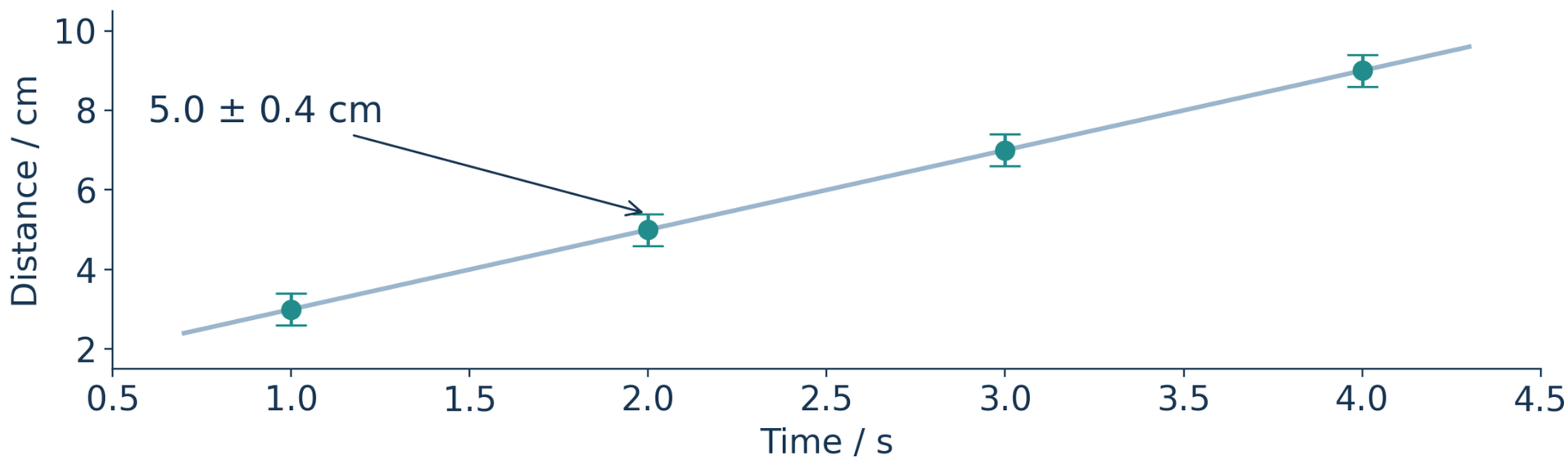
$$\text{rate} = (0.40 \pm 0.02)\ \text{mL}\,\text{s}^{-1}$$

Put the uncertainty back in units

5.0% of 0.400 is 0.020. Rate = 0.40 ± 0.02 mL s^{-1}. Repeats with independent biological samples can also reveal real variation between samples.

Find the operation first: a difference needs absolute uncertainties; a rate needs percentages.

Why draw error bars?



A dot looks more exact than it is

A point at 5.0 cm with ±0.4 cm uncertainty has a bar from 4.6 to 5.4 cm. The bar makes the stated range visible.

Label what the bar means

A bar could show a reading estimate, SD or SE. These answer different questions. Include the unit and say what the bars represent.

Compare with care

Overlapping stated ranges can show that values are compatible with those ranges. They do not prove the values are equal. For statistical bars, overlap alone is not a general significance test.

The bar is useful only when the reader knows what it represents.

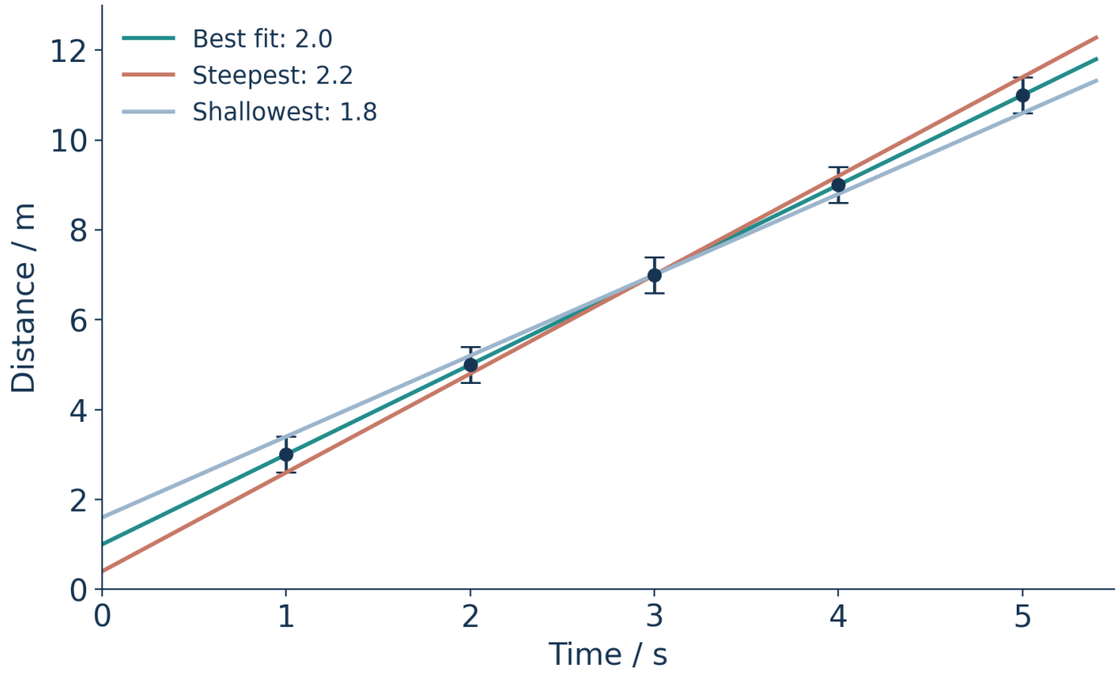
Why can a graph have more than one slope?

The points allow some movement

If measurements have uncertainty, more than one straight line may fit their bars. A steeper or shallower line gives a different gradient.

Use all the bars

Find the steepest and shallowest acceptable lines. Here their gradients are 2.2 and 1.8 m s^{-1}. Half the difference is 0.2 m s^{-1}, so the best-fit gradient is 2.0 ± 0.2 m s^{-1}.



$$\text{gradient uncertainty} = \frac{2.2 - 1.8}{2} = 0.2\ \text{m s}^{-1}$$

The intercept can move too

The same lines cross the vertical axis at 0.4 and 1.6 m. The best-fit intercept is 1.0 ± 0.6 m. A non-zero intercept may suggest an offset if theory predicts zero, but it does not identify the cause.

A gradient is calculated from uncertain data, so it also needs an uncertainty.

Your turn: explain the reason as well

A. A temperature rise

Water warms from 20.0 ± 0.1 °C to 28.0 ± 0.1 °C. Find the rise and its uncertainty. Why do you add the uncertainties?

B. Energy per gram

A sample transfers 600 J with 3% uncertainty. Mass burned is 2.00 g with 1% uncertainty. Find the energy per gram and its absolute uncertainty.

C. What can you conclude?

A result is 40% below a suitable reference but has 2% calculated reading uncertainty. Does that prove the trials were repeatable? What else should you investigate?

D. Which measurement matters?

For density, mass contributes 0.5% uncertainty and volume contributes 3.0%. Which contribution should you reduce first, and why?

Try the calculations first, then say what each result means.

Answers: the reasoning behind the numbers

A. The rise is 8.0 ± 0.2 °C

28.0 − 20.0 = 8.0 °C. Both readings can change the gap, so add 0.1 + 0.1 = 0.2 °C. A high final and low initial reading make the rise larger.

B. Energy per gram is 300 ± 12 J g^{-1}

600 ÷ 2.00 = 300 J g^{-1}. Division combines percentage uncertainties: 3% + 1% = 4%. Four per cent of 300 is 12 J g^{-1}. Here we retain two uncertainty digits.

C. Repeatability needs repeat data

The 2% comes from the readings included in the calculation. It does not show how repeated trials varied. Investigate heat losses, calibration, other omitted effects and whether the reference is suitable.

D. Start with the volume term

Volume contributes more. Halving it reduces the total from 3.5% to 2.0%. Halving only the mass contribution reduces the total to 3.25%. Explain how your chosen method improves volume measurement.

A useful answer gives the result, the rule and the reason.